

Quintessential Phenomena in Higher Dimensional Space Time

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Abstract

The higher dimensional cosmology provides a natural setting to treat, at a classical level, the cosmological effects of vacuum energy. Here we discuss two situations where starting with an ordinary matter field without any equation of state we end up with a Chaplygin type of gas apparently as a consequence of extra dimensions. In the second case we study the quintessential phenomena in higher dimensional spacetime with the help of a Chaplygin type of matter field. The first case suffers from the disqualification that no dimensional reduction occurs, which is, however, rectified in the second case. Both the models show the sought after feature of occurrence of *flip* in the rate of expansion. It is observed that with the increase of dimensions the occurrence of *flip* is delayed for both the models, more in line with current observational demands. Interestingly we see that depending on some initial conditions our model admits QCDM, Λ CDM and also Phantom like evolution within a unified framework. Our solutions are general in nature in the sense that when the extra dimensions are switched off the known 4D model is recovered. Correspondence to a recent work of Guo et al for a *quiescence* like model is also found.

KEYWORDS : cosmology; higher dimensions; accelerating universe

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1. Introduction

Distance measurement of type Ia supernovae as well as cosmic microwave background anisotropy measurements in the last decade points to a late accelerated expansion of the universe. Gravitational interaction being always attractive in nature this finding, corroborated by a number of cosmic probes has been puzzling astronomers for a long time and has so far evaded any plausible but consistent explanation based on sound physical principles. Briefly stated - proposals include among others flavor oscillations of axions [1], modification of Einstein-Hilbert action via the additional curvature terms in the Lagrangian [2], introduction of an evolving cosmological constant in the field equations, the role of inhomogeneity to cause acceleration [3], Brans-Dicke scalar field

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and finally the presence of a quintessential type of scalar field giving rise to a large negative pressure(dark energy) [4].

In this context the authors of the present article have been, of late, struggling with the idea of explaining the late acceleration as a higher dimensional(HD) effect [5, 6]. In the framework of higher dimensional cosmology we have been able to show, though in a rather naive way, that the acceleration can be explained as a consequence of the presence of the extra spatial dimensions and this effect has been coined as ‘*dimension driven*’ accelerating model. In fact here the effective Friedmann equations contain additional terms coming from the extra dimensions which may be viewed as a ‘fluid’ responsible for the current acceleration. So here we attempt to incorporate the phenomenon of acceleration within the framework of HD spacetime itself without invoking a mysterious scalar field with large negative pressure by hand. On the contrary the origin of the extra fluid responsible for the acceleration is geometric in origin having strong physical foundation and more in line with the spirit of general relativity as proposed by Einstein [7] and later developed by Wesson and his collaborators[8]. Moreover in a recent communication [9] it is argued that quantum fluctuation in 4D spacetime do not give rise to dark energy but rather a possible source of the dark energy is the fluctuations in the quantum fields including quantum gravity inhabiting extra compactified dimensions. So we recently witness a spurt of activities relating to higher dimensional spacetime in its attempts to unify gravity with other forces of nature, development of varied brane models, induced matter proposal and very recently the dimension driven quintessential models. The present investigation is primarily motivated by two considerations. While we have plenty of multidimensional cosmological models in literature [10] and also some sporadic works of Chaplygin inspired brane models [11, 12] we are not much aware of models of similar kind in higher dimensions either driven by extra dimensions themselves or by Chaplygin type of gas.

The present work essentially comprises two parts. We have taken a $(d+4)$ dimensional homogeneous spacetime with two scale factors and a perfect fluid as a source field. Assuming an ansatz in the form of the deceleration parameter we find an expression of the scale factor for isotropic expansion. In the process we get a class of solutions for the matter field with interesting physical properties closely mimicing the well known expression of a generalised Chaplygin gas suitable for accelerating model. One may look upon the resulting matter field as a two-component fluid consisting a cosmological constant and a perfect fluid obeying a higher dimensional equation of state. One finds that in the early phase the pure matter field predominates resulting in a decelerated expansion followed by the predominance of the cosmological constant with the expansion transiting to an accelerating phase, while in the intermediate stages our cosmology interpolates between the different phases of the universe. For sake of completeness we also construct an equivalent scalar field with its potential energy term to simulate the dynamics of the cosmology with the actual matter field.

In the second part of the program the approach is reversed. Here we start with the more conventional approach of taking a Chaplygin type of matter field to start with. Our first model suffers from the disqualification that both 4D and HD scale factors are

expanding so the final reduction to the current manifestly 4D universe does not work. This shortcoming is remedied in the second part. While literature abounds with quintessential models with Chaplygin type of gas in 4D we are not aware of results of similar kind in HD spacetime. However we have not been able to find solutions in a closed form with the system of equations finally reducing to a hypergeometric series. In any case certain inferences can always be drawn in the extreme cases and our analysis shows that an initially decelerating model transits to an accelerating one as in 4D. An interesting result in this section is the fact that the effective equation of state at the late stage of evolution contains some additional term coming from extra dimensions. Fixing some initial conditions the cosmology evolves as QCDM, Λ CDM or Phantom type. Though not exactly similar this points to the ‘*k-essence*’ type of models which lead to cosmic acceleration today for a wide range of initial conditions without fine-tuning and without invoking an anthropic argument.

2. Accelerating Universe - I

We consider the line element of (d+4)-dimensional spacetime

$$ds^2 = dt^2 - R^2 \left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) - A^2 \gamma_{ab} dy^a dy^b \quad (1)$$

where y^a ($a, b = 4, \dots, 3+d$) are the extra dimensional spatial coordinates and the 3D and extra dimensional scale factors R and A depend on time only and K is the 3D curvature and the compact manifold is described by the metric γ_{ab} . For our manifold $M^1 \times S^3 \times S^d$ the symmetry group of the spatial section is $O(4) \times O(d+1)$. The stress tensor whose form will be dictated by Einstein’s equations must have the same invariance leading to the energy momentum tensor as [13]

$$T_{00} = \rho, \quad T_{ij} = -p(t)g_{ij}, \quad T_{ab} = -p_d(t)g_{ab} \quad (2)$$

where the rest of the components vanish. Here p is the isotropic 3-pressure and p_d , that in the extra dimensions.

The independent field equations for our metric (1) are

$$\rho = 3 \frac{\dot{R}^2}{R^2} + \frac{1}{2} d(d-1) \frac{\dot{A}^2}{A^2} + 3d \frac{\dot{R}\dot{A}}{RA} \quad (3)$$

$$p = -2 \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - d \frac{\ddot{A}}{A} - \frac{1}{2} d(d-1) \frac{\dot{A}^2}{A^2} - 2d \frac{\dot{R}\dot{A}}{RA} \quad (4)$$

$$p_d = -3 \frac{\ddot{R}}{R} - 3 \frac{\dot{R}^2}{R^2} - (d-1) \frac{\ddot{A}}{A} - \frac{1}{2} (d-1)(d-2) \frac{\dot{A}^2}{A^2} - 3(d-1) \frac{\dot{R}\dot{A}}{RA} \quad (5)$$

Here we have three independent equations with five unknowns (ρ, p, p_d, A, R) and hence we are at liberty to conveniently fix up two initial conditions. Firstly we take a perfect fluid with isotropic pressure in all dimensions such that $p = p_d$ and secondly we assume a particular form of deceleration parameter as

$$q = \frac{a - R^w}{b + R^w} \quad (6)$$

where a, b and w are arbitrary constants. The equation (6) yields

$$R(t) = R_0 \sinh^n \omega t \quad (7)$$

($n = \frac{2}{w}$) such that we get

$$q = \frac{1 - n \cosh^2 \omega t}{n \cosh^2 \omega t} \quad (8)$$

showing that the exponent n determines the evolution of q . While for $n > 1$, it is only accelerating but for $n < 1$ we are able to achieve the desirable feature of *flip*, although it is not obvious from our analysis at what value of redshift this *flip* occurs. The second assumption $p = p_d$ gives

$$-\frac{\ddot{R}}{R} - 2\frac{\dot{R}^2}{R^2} + \frac{\ddot{A}}{A} + (d-1)\frac{\dot{A}^2}{A^2} + (3-d)\frac{\dot{R}\dot{A}}{RA} = 0 \quad (9)$$

Relevant to point out that $R = A$ identically satisfies this equation and in this case the matter density and isotropic pressure come out to be

$$\rho = \frac{1}{2}(d+2)(d+3)\frac{\dot{R}^2}{R^2} \quad (10)$$

$$p = p_d = -(d+2)\frac{\ddot{R}}{R} - \frac{1}{2}(d+1)(d+2)\frac{\dot{R}^2}{R^2} \quad (11)$$

Now we are in a position to calculate the expressions for matter density and pressure and find

$$\rho = \Lambda + \frac{B}{R^{\frac{2}{n}}} \quad (12)$$

$$p = p_d = -\Lambda - \frac{n(d+3) - 2}{n(d+3)} \frac{B}{R^{\frac{2}{n}}} \quad (13)$$

where, $\Lambda = \frac{1}{2}(d+2)(d+3)n^2\omega^2$ and $B = \Lambda R_0^{\frac{2}{n}}$.

We also get the equation of state as

$$p = (\gamma - 1)\rho - \frac{2\Lambda}{(d+3)n} = (\gamma - 1)\rho - \gamma\Lambda \quad (14)$$

where, $(\gamma - 1) = -\frac{(d+3)n-2}{(d+3)n}$, so $n = \frac{2}{\gamma(d+3)}$ and $\omega = \gamma\sqrt{\frac{(d+3)\Lambda}{2(d+2)}}$ such that we finally get

$$R(t) = R_0 \left[\sinh \sqrt{\frac{(d+3)\Lambda}{2(d+2)}} \gamma t \right]^{\frac{2}{\gamma(d+3)}} \quad (15)$$

The last equation makes interesting reading. As $t \sim 0$, $\sinh t \sim t$ and we get

$$R = t^{\frac{2}{\gamma(d+3)}} \quad (16)$$

such that for 4D case ($d = 0$) [14]

$$R = t^{\frac{2}{3\gamma}} \quad (17)$$

which yields the well known solutions as $t = t^{\frac{1}{2}}$, $t = t^{\frac{2}{3}}$ for radiation and dust respectively for 4D. Replacing n in the matter field equations we finally get

$$\rho = \Lambda + \frac{B}{R^{\gamma(d+3)}} \quad \text{and} \quad p = p_d = -\Lambda - (1 - \gamma) \frac{B}{R^{\gamma(d+3)}} \quad (18)$$

From our analysis it follows that

(i) for dust, $\gamma = 1$ and $n = \frac{2}{d+3}$

$$\rho = \Lambda + \frac{B}{R^{(d+3)}} \quad \text{and} \quad p = -\Lambda \quad (19)$$

(ii) for radiation, $\gamma = \frac{d+4}{d+3}$ and $n = \frac{2}{d+4}$

$$\rho = \Lambda + \frac{1}{R^{(d+4)}} \quad \text{and} \quad p = -\Lambda + \frac{1}{(d+3)} \frac{B}{R^{(d+4)}} \quad (20)$$

(iii) for stiff fluid, $\gamma = 2$ and $n = \frac{1}{d+3}$

$$\rho = \Lambda + \frac{B}{R^{2(d+3)}} \quad \text{and} \quad p = -\Lambda + \frac{B}{R^{2(d+3)}} \quad (21)$$

One can calculate q for different values of γ . Taking

(i) $\gamma = 2$ (stiff fluid)

$$q = \frac{(d+2) - \sinh^2 \omega t}{1 + \sinh^2 \omega t} \quad (22)$$

(ii) $\gamma = \frac{d+4}{d+3}$ (radiation)

$$q = \frac{(d+2) - 2 \sinh^2 \omega t}{2 + 2 \sinh^2 \omega t} \quad (23)$$

(iii) $\gamma = 1$ (dust)

$$q = \frac{(d+1) - 2 \sinh^2 \omega t}{2 + 2 \sinh^2 \omega t} \quad (24)$$

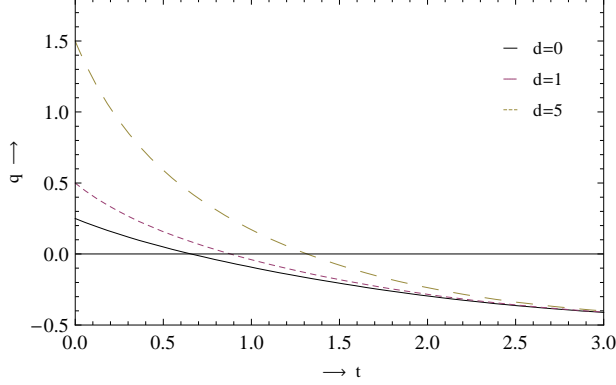


Figure 1: The variation of q and t for different values of d in dust dominated era. *Flip* is delayed as dimensions increases.

At this stage one can check the influence of extra dimension on the occurrence of *flip*. The figure-1 shows that with dimensions the instant of *flip* is delayed. The fact that the time of *flip* is delayed with dimensions is in line with observational demands. This is because there is plenty of observational evidence for a decelerated universe in the recent past [15]. However the dominance of vacuum energy over ordinary matter at some $z_{eq} > 0$ does not guarantee the present acceleration of the universe. For this, the vacuum energy has to dominate long enough as to overcome the gravitational attraction produced by ordinary matter. Current dynamical mass measurements suggest that the matter content of the universe adds up to 30 percent of the critical density. It is generally believed that the transition from deceleration to acceleration occurs at a redshift $0.28 < z_T < 0.68$, which confirms the idea that the acceleration is a recent phenomenon. Hence the significance of the above plot is particularly encouraging in the context of higher dimensional cosmology.

Now the expression of speed of sound will be

$$C_s^2 = \frac{\delta p}{\delta \rho} = \frac{2}{\gamma} - 1 \quad (25)$$

which gives (i) for dust case: $C_s^2 = 0$; (ii) for radiation case : $C_s^2 = \frac{1}{d+3}$ and (iii) for stiff fluid : $C_s^2 = 1$. Thus we see that the velocity of sound vanishes in the dust model as expected and it is maximum in 4D and decreases with dimension in radiation dominated era. To avoid imaginary value of the speed of the sound $\gamma < 2$. Again C_s should never exceed the speed of light for the range of γ , $1 < \gamma < 2$.

As is well known the whole dynamics can be simulated in a field theoretic approach with the help of an equivalent scalar field [16]. For a scalar field ϕ having self-interacting po-

tential $U(\phi)$, the Lagrangian of the scalar field will be $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - U(\phi)$, the equivalent energy density ρ_ϕ and pressure p_ϕ for the scalar field will be

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + U(\phi) = \Lambda + \frac{B}{R^{\gamma(d+3)}} \quad (26)$$

and the corresponding ‘pressure’ as

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - U(\phi) = -\Lambda - \frac{B}{R^{\gamma(d+3)}} + \gamma \frac{B}{R^{\gamma(d+3)}} \quad (27)$$

such that

$$\dot{\phi}^2 = \gamma \frac{B}{R^{\gamma(d+3)}} \quad (28)$$

which, in turn, gives via equation (3) for (d+4) spacetime

$$\phi' = \pm \sqrt{\frac{B\gamma(d+2)(d+3)}{2}} \frac{1}{R\sqrt{\Lambda R^{\gamma(d+3)} + B}} \quad (29)$$

(here a prime and a dot overhead denote differentiation w.r.t. R and t respectively.)

$$\phi = \sqrt{\frac{2(d+2)}{\gamma(d+3)}} \tanh^{-1} \left(\sqrt{\frac{\Lambda R^{\gamma(d+3)} + B}{B}} \right) \quad (30)$$

or

$$\phi = \sqrt{\frac{2(d+2)}{\gamma(d+3)}} \tanh^{-1} (\cosh \omega t) \quad (31)$$

and

$$U(\phi) = \frac{\Lambda\gamma}{2} \left\{ 1 + \left(1 - \frac{2}{\gamma} \right) \sinh^2 \sqrt{\frac{1 + (1 - \frac{2}{\gamma})(d+3)}{2(d+2)}} \phi \right\} \quad (32)$$

We also see from figure-2 that for dust and radiation cases $U(\phi)$ decreases with ϕ unlike the stiff fluid case where it remains constant.

It may not be out of place to call attention to a quintessential model driven by a tachyonic scalar field [17] with a potential in 4D space time

$$V(T) = \frac{\Lambda}{\sin^2 \left(\frac{3\sqrt{\Lambda\gamma}}{2} \right) T} \sqrt{1 - \gamma \cos^2 \left(\frac{3\sqrt{\Lambda\gamma}}{2} \right) T} \quad (33)$$

(T is a tachyonic scalar field) giving the cosmological evolution as

$$R(t) = R_0 \left(\sinh \frac{3\sqrt{\Lambda\gamma}t}{2} \right)^{\frac{2}{3\gamma}} \quad (34)$$

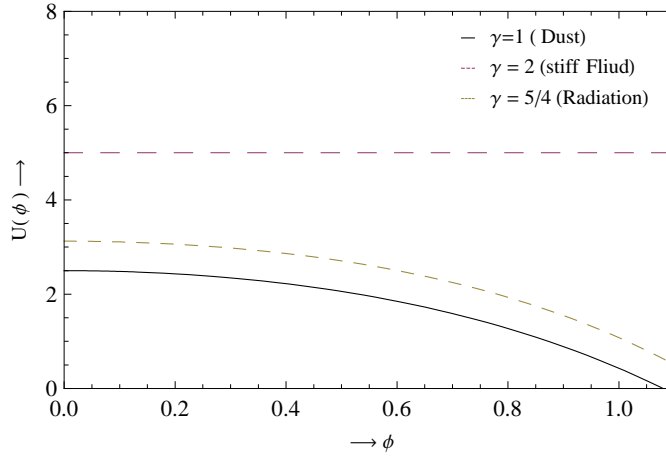


Figure 2: The variation of $U(\phi)$ and ϕ is shown in this figure for different values of γ in 5D case.

It behaves like a two fluid model where one of the fluids is a cosmological constant while the other obeys a state equation $p = (\gamma - 1)\rho$, ($0 < \gamma < 2$). Similarity of this evolution with our model is more than apparent except for some numerical factors coming out because we are here dealing with a higher dimensional spacetime. But the main result may be re-emphasized that we get this evolution without forcing ourselves to invoke any extraneous tachyonic type of scalar field.

3. Accelerating Universe - II

As commented in the introduction the first model suffers from the shortcoming that no dimensional reduction occurs. We here adopt the conventional approach of taking a predetermined form of energy momentum tensor and then try to solve the equations of motion with the gauge condition

$$A(t) = R(t)^{-m} \quad (35)$$

where m is any positive number so that dimensional reduction is ensured *a priori*. For the matter field we here assume an equation of state of a generalised Chaplygin type of gas in 3D space only

$$p = (\gamma - 1)\rho - B\rho^{-\alpha} \quad (36)$$

where $0 \leq \alpha \leq 1$ along with a higher dimensional isotropic pressure p_d existing in the

extra space such the the field equations reduce to

$$\rho = \frac{k}{2} \frac{\dot{R}^2}{R^2} \quad (37)$$

$$-p = (2 - dm) \frac{\ddot{R}}{R} + \frac{1}{2} [m^2 d(d+1) + 2(1 - dm)] \frac{\dot{R}^2}{R^2} \quad (38)$$

$$-p_d = (3 - dm + m) \frac{\ddot{R}}{R} + \frac{1}{2} [m(d-1)(dm-4) + 6] \frac{\dot{R}^2}{R^2} \quad (39)$$

where $k = dm^2(d-1) + 6(1-dm)$.

For positive energy density, k must be greater than zero which gives $m < \frac{3d-\sqrt{3d(d+2)}}{d(d-1)}$ or, $m > \frac{3d+\sqrt{3d(d+2)}}{d(d-1)}$.

Again Bianchi identity will be in our case

$$\dot{\rho} + 3 \frac{\dot{R}}{R} (\rho + p) + d \frac{\dot{A}}{A} (\rho + p_d) = 0 \quad (40)$$

Now using equations (36), (37) & (40) we get

$$\dot{\rho} + \frac{k}{(2-dm)} \frac{\dot{R}}{R} \left[\left\{ \gamma + \frac{2dm(m+1)}{k} \right\} \rho - B \rho^{-\alpha} \right] = 0 \quad (41)$$

Solving equation (41) we get

$$\rho = \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{1}{1+\alpha}} \quad (42)$$

where

$$M = \gamma k + 2dm(m+1) \quad (43)$$

and c is the integration constant. From physical considerations we can finally put the restriction on m as $m < \frac{3d-\sqrt{3d(d+2)}}{d(d-1)}$ for $d \neq 1$ and $m < 1$ for $d = 1$.

The relation (43) needs some interpretation. The last term is a typical higher dimensional effect, absent in 4D ($d = 0$). Now we see that several possibilities for different values of m present themselves.

(i) $m = -1$: Here $A = R$ and we have to sacrifice the desirable feature of dimensional reduction to get an isotropic expansion in all dimensions. However it not exactly the

cosmology of the section I because we have a Chaplygin type of gas to start with in this case. Moreover the isotropy of the metric dictates that $p = p_d$ and so we end up with an isotropic pressure in all dimensions. The solutions closely resemble the earlier work of Debnath *et al* [18] in 4D.

(ii) $m = 0$: Here we get flat extra space although the total number of dimensions continues to be $(d + 4)$. But the cosmology is exactly similar to the 4D case referred to earlier [18]. In fact this similarity is a direct consequence of a little known theorem of Campbell that any analytic N-dimensional Riemmanian manifold can be locally embedded in a higher dimensional Ricci-flat manifold [19].

(iii) $d = 0$: Here we simply recover the 4D metric and all the known solutions of 4D follow.

Now with the help of equations (37) & (42), we get

$$\frac{\dot{R}^2}{R^2} = \frac{2}{k} \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{1}{1+\alpha}} \quad (44)$$

We have not been able, so far, to find a solution of equation (44) in a closed form. Rather a Hypergeometric series solution results given by

$$2(2 - dm) {}_2F_1 \left[s, s, 1 + s, -\frac{BkR^{\frac{M}{2s(2-dm)}}}{cM} \right] R^{\frac{M}{2(2-dm)}} = \frac{\sqrt{2}Mc^st}{\sqrt{k}} + C \quad (45)$$

where $s = \frac{1}{2(1+\alpha)}$ and ${}_2F_1$ is the hypergeometric function.

Even then, fixing the values of different parameters one can get the temporal behaviour of the scale factors as given in the adjoining figure-3. A cursory look at the figure shows that at a certain stage of evolution the cosmology starts inflating. Another desirable feature is the fact that the extra dimensions compactify at very early stage of evolution in conformity with both theoretical and observational requirements.

Now for small value of scale factor $R(t)$ ($A(t)$ should be large in this situation),

$$\rho \approx c^{\frac{1}{1+\alpha}} R^{-\frac{M}{(2-dm)}} \quad (46)$$

which is very large and corresponds to the universe dominated by an equation of state, $p = (\gamma - 1)\rho$ as is evident from equation (36). At the late stage of evolution (when $R(t)$

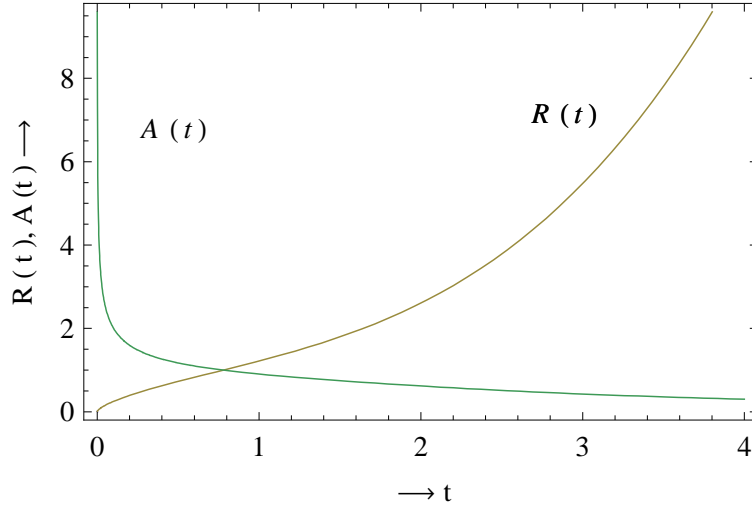


Figure 3: The variation of $R(t)$ and $A(t)$ vs t is shown in this figure. Here we see that $R(t)$ expands indefinitely and dimensional reduction is possible for extra dimensions. We have taken $\alpha = 0.5$, $m = 0.5$, $d = 1$ and $\gamma = 1$. This graph is drawn with the help of 'Mathematica' using equations (35) & (44).

is large and $A(t)$ small) the expressions for ρ and p are given below.

$$\begin{aligned} \rho &\approx \left(\frac{Bk}{M}\right)^{\frac{1}{1+\alpha}} \text{ and } p \approx -\left(\frac{Bk}{M}\right)^{\frac{1}{1+\alpha}} \left[\frac{M}{k} - (\gamma - 1)\right] = -\left[\frac{M}{k} - (\gamma - 1)\right] \rho \\ &= -\left[1 + \frac{2dm(m+1)}{k}\right] \rho = \mathcal{W}\rho \end{aligned} \quad (47)$$

where

$$\mathcal{W} = -\left[1 + \frac{2dm(m+1)}{k}\right] \quad (48)$$

The last equation is interesting and introduces some new physics in our analysis in the context of multidimensional cosmology. Here $\mathcal{W} \neq -1$. Obviously this is due to the presence of extra dimensions in the above equation. As mentioned in the introduction we have been working on the idea of 'dimension driven' acceleration for the past few years and the expression (47) points to the fact - how extra dimensions aid the inflationary process.

In 4D case ($d = 0$) $\mathcal{W} = -1$ and a Λ CDM model results. Otherwise the magnitude of \mathcal{W} is parameter dependent. When $m = 0$, i.e. $A(t)$ is a constant we again get back the 4D case. This is, however, a consequence of the Campbell's theorem [19] as mentioned earlier.

When $m > 0$, $\mathcal{W} < -1$; So a phantom like cosmology results with the occurrence of ‘big rip’ etc. But the cosmology becomes physically interesting when $-1 < m < 0$ such that $0 > \mathcal{W} > -1$ and we get a quiescence type of model [20]. In this connection one should note that predictions from current measurements illustrate a degeneracy in which a flat Λ CDM model is indistinguishable from closed Λ CDM model, although the predicted value of \mathcal{W} based on CMB measurement is about $\mathcal{W} = -\frac{2}{3}$. However, the error bars are too broad to reach firm conclusions.

It will be not out of space to call attention to a recent work by Guo and Zhang et al[21] where a very generalised form of Chaplygin relation is invoked where the constant B is assumed to depend on the scale factor *i.e.*, $B = B(a)$. Taking $B = B_0 a^{-n}$, for example, it is shown that for a very large value of the scale factor the model interpolates between a dust dominated phase and a *quiescence-dominated phase* (*i.e.*, dark energy with a constant equation of state)[22] given by $W = -1 + n/6$. So our model closely mimics the above work where the extra dimensions takes the role of a variable $B(a)$.

Now, if we calculate (3+d)-dimensional volume

$$V = R^3 A^d = R^{(3-dm)} \quad (49)$$

Since $m < \frac{3d - \sqrt{3d(d+2)}}{d(d-1)}$ for $d \neq 1$ and $m < 1$ for $d = 1$, \therefore the value of $(3 - dm)$ should be positive.

Now for accelerating universe, $\ddot{R} > 0$, implying

$$R^{\frac{M(1+\alpha)}{(2-dm)}} > \frac{c}{2B} \frac{M(M + 2dm - 4)}{k(2 - dm)} \quad (50)$$

The above expression shows that the universe will be decelerating for small values of scale factor while for large values we get accelerating universe and the flip occurs at $t = t_c$ such that

$$R(t_c) = \left[\frac{c}{2B} \frac{M(M + 2dm - 4)}{k(2 - dm)} \right]^{\frac{2-dm}{M(1+\alpha)}} \quad (51)$$

While looking at the plot (figure-4) one must admit that it is rather artificial to take the number of spatial dimensions as a continuous variable. However given the fact that in Friedmannian cosmology $R(t)$ is always a monotonically increasing function of time, $R(t)$ may be taken as a measure of the flipping instant and incidentally it does not initially show much variation with d . The fact that it shoots up at some higher value of d need not be taken too seriously because it may be due to the arbitrarily chosen initial value of α , m etc. In any case the *flip* is delayed with dimensions and from observational point it is encouraging because all the evidences from different cosmic probes point to a very late acceleration.

The speed of sound $v_s = \frac{\partial p}{\partial \rho}$ in the Chaplygin gas

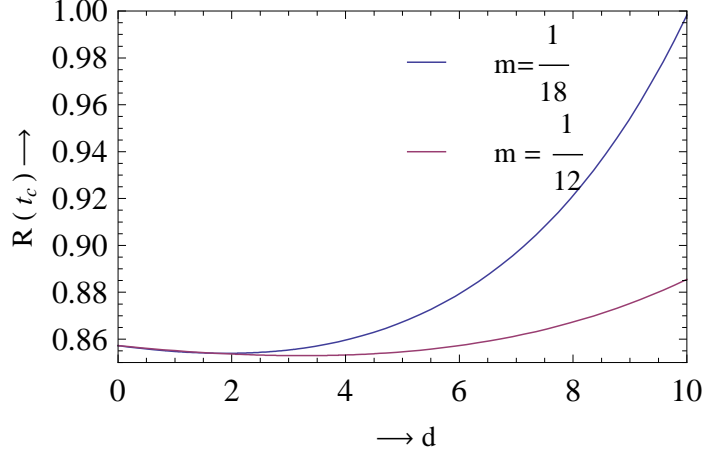


Figure 4: The variation of $R(t_c)$ and d is shown in this figure for different values of m . Here we have taken $\alpha = 0.5$ and $\gamma = 1$.

$$v_s^2 = (\gamma - 1)(1 + \alpha) - \frac{\alpha p}{\rho} \quad (52)$$

Using equations (38), (39) & (52) we get,

$$v_s^2 = \frac{1 + \alpha}{d + 3} + \frac{\alpha}{k} \{m^2 d(d + 1) + 2(1 - dm)\} + \frac{2\alpha q}{k}(2 - dm) \quad (53)$$

where q is the deceleration parameter defined as $-\frac{\ddot{R}R}{\dot{R}^2}$. For accelerating model the value of q should be negative. So for real value of v_s , $\frac{1+\alpha}{d+3} + \frac{\alpha}{k} \{m^2 d(d+1) + 2(1-dm)\} > \frac{2\alpha|q|}{k}(2-dm)$ which gives $\frac{k(1+\alpha)}{2\alpha(d+3)(2-dm)} + \frac{1}{2(2-dm)} \{m^2 d(d+1) + 2(1-dm)\} > |q|$. This relation gives an upperbound for the q , which may have observational consequences. Since $0 < \alpha < 1$, it can not exceed the speed of light. But for larger values of α the speed of sound exceeds the speed of light. It also depends upon the value of m . In cosmology a speed of sound exceeding the speed of light does not contradict the causality [23].

Let $\alpha = 0.5$, $m = \frac{1}{6}$, and $d = 6$, therefore k should be $\frac{5}{6}$ and M equal to $\frac{88}{27}$ in this case. If we calculate v_s^2 , we get two conditions (i) when $|q| < 0.923$ then $v_s < c$, (ii) otherwise $v_s > c$.

Following the analysis given in the previous section we get the analogous energy density ρ_ϕ and pressure p_ϕ for the scalar field as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + U(\phi) = \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{1}{1+\alpha}} \quad (54)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - U(\phi) = (\gamma - 1)\rho - B\rho^{-\alpha} \quad (55)$$

$$= (\gamma - 1) \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{1}{1+\alpha}} - B \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{\alpha}{1+\alpha}} \quad (56)$$

such that we get

$$\dot{\phi}^2 = \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{-\frac{\alpha}{1+\alpha}} \left[\left(\gamma \frac{Bk}{M} - B \right) + \frac{c\gamma}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right] \quad (57)$$

$$\phi' = \pm \frac{1}{R} \sqrt{\frac{k}{2}} \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{-\frac{1}{2}} \left[B \left(\gamma \frac{k}{M} - 1 \right) + \frac{c\gamma}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{\frac{1}{2}} \quad (58)$$

and

$$U(\phi) = \frac{1}{2}(2 - \gamma) \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{-\frac{1}{1+\alpha}} + \frac{B}{2} \left[\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}} \right]^{-\frac{\alpha}{1+\alpha}} \quad (59)$$

Integrating equation (58)

$$\phi - \phi_0 = \pm \int \left[\frac{1}{R} \sqrt{\frac{k}{2}} \left\{ \frac{B \left(\gamma \frac{k}{M} - 1 \right) + \frac{c\gamma}{R^{\frac{(1+\alpha)M}{(2-dm)}}}}{\frac{Bk}{M} + \frac{c}{R^{\frac{(1+\alpha)M}{(2-dm)}}}} \right\} \right]^{\frac{1}{2}} dR \quad (60)$$

where ϕ_0 is an arbitrary constant of integration. It is very difficult to integrate the equation (60) in a closed form. However if at this stage we take the $m = -1$ i.e., $\frac{\gamma k}{M} = 1$ then we are able to integrate and get the following solution.

$$\phi = \sqrt{\frac{2}{k\gamma}} \frac{2+d}{1+\alpha} \sinh^{-1} \left\{ \sqrt{\frac{c\gamma}{B}} R^{-\frac{(1+\alpha)k\gamma}{2(2+d)}} \right\} \quad (61)$$

and

$$U(\phi) = \frac{2-\gamma}{2} \left(\frac{B}{\gamma} \right)^{\frac{1}{1+\alpha}} \cosh^{\frac{2}{1+\alpha}} \left\{ \sqrt{\frac{k\gamma}{2}} \frac{1+\alpha}{d+2} \phi \right\} \\ + \frac{B}{2} \left(\frac{B}{\gamma} \right)^{-\frac{\alpha}{1+\alpha}} \cosh^{-\frac{2\alpha}{1+\alpha}} \left\{ \sqrt{\frac{k\gamma}{2}} \frac{1+\alpha}{d+2} \phi \right\} \quad (62)$$

If we put $d = 0$, i.e., in 4-Dimensional spacetime we get solutions of the above equations (61) & (62) which are identical with 4D case [18].

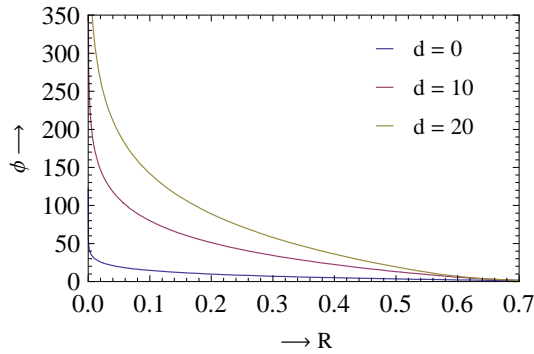


Figure 5: The variation of ϕ and R is shown in this figure. The scalar field decays with time more sharply in lower dimensions. (Taking $\alpha = 0.6$, $k = 6$ and $\gamma = 1$).

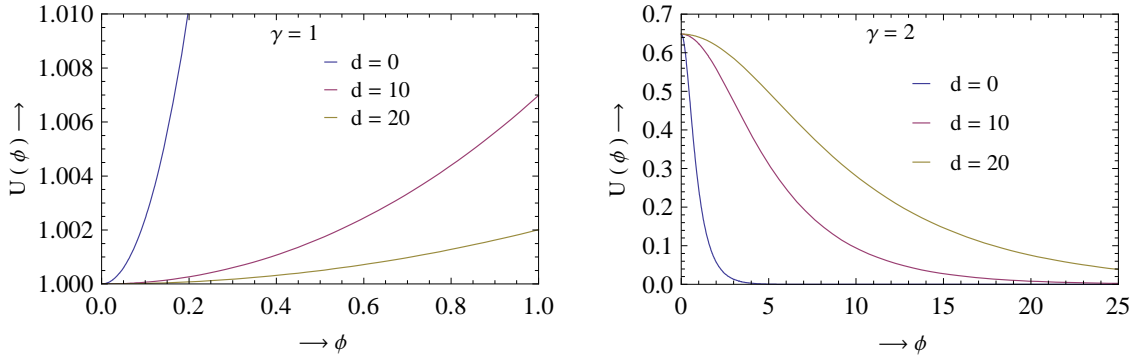


Figure 6: The variation of $U(\phi)$ and ϕ is shown. It suggests critical γ dependance. With dimensions $U(\phi)$ becomes flatter. ($\alpha = 0.6$, $k = 6$).

4. Discussion

We have here studied two higher dimensional cosmological models amenable to late acceleration - one with a particular form of deceleration parameter and for the other with a generalised chaplygin type of gas. The important findings may be briefly summarised as:

(i) Unlike the first case we have dimensional reduction in the second.

(ii) The striking thing about the first model is the fact that we do not have to assume an extraneous, unphysical quintessential type of matter field *a priori* to achieve the late acceleration.

(iii) It is observed that the instant of *flip* is delayed with dimensions, which is also supported by current observations. In this respect the HD accelerating models have an edge over the 4D ones.

(iv) The most important thing in the second case, in our opinion, is the finding that

depending on some initial conditions, the effective equation of state during late evolution interpolates among Λ CDM, QCDM and Phantom type of expansion. In this respect our work recovers the effective equation of state (for large scale factor) for a recent work of Guo et al where a very generalised Chaplygin type of gas is taken.

(v) Our solutions are general in nature because all the known results of 4D cosmology are recovered when $d = 0$.

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